LINEAR OPTIMIZATION IN APPLICATIONS

S.L. TANG

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1 INTRODUCTION

1.1 Formulation of a Linear Programming Problem

Linear programming is a powerful mathematical tool for the optimization of an objective under a number of constraints in any given situation. Its application can be in maximizing profits or minimizing costs while making the best use of the limited resources available. Because it is a mathematical tool, it is best explained using a practical example.

Example 1.1

A pipe manufacturing company produces two types of pipes, type I and type II. The storage space, raw material requirement and production rate are given as below:

Resources	Type I	Type II	Company Availability
Storage space	5 m^2/pipe	3 m^2/pipe	750 m^2
Raw materials	6 kg/pipe	4 kg/pipe	800 kg/day
Production rate	30 pipes/hour	20 pipes/hour	8 hours/day

The profit for selling one type I pipe is \$10 and that for type II is \$8. The pipes produced each day are taken by trucks to sales outlets in the early morning of the next day before a new day's manufacturing work starts. Our objective is to formulate for the company a linear programming model which can determine how many pipes of each type should be manufactured each day so that the total profit can be maximized.

Solution 1.1

Let Z = total profit

x_1 = number of type I pipes produced each day

x_2 = number of type II pipes produced each day

Since our objective is to maximize profit, we write an **objective function**, equation (0), which will calculate the total profit:

$$\text{Maximize } Z = 10x_1 + 8x_2 \quad \text{———} \quad (0)$$

x_1 and x_2 in equation (0) are called **decision variables**.

There are three constraints which govern the number of type I and type II pipes produced. These constraints are: (1) the availability of storage space, (2) the raw materials available, and (3) the working hours of labourers. Constraints (1), (2) and (3) are written as below:

Storage space : $5x_1 + 3x_2 \leq 750$ ———————— (1)

Raw material : $6x_1 + 4x_2 \leq 800$ ———————— (2)

Working hours : $\frac{x_1}{30} + \frac{x_2}{20} \leq 8$ ———————— (3)

When constraint (3) is multiplied by 60, the unit of hours will be changed to the unit of minutes (ie. 8 hours to 480 minutes). Constraint (3) can be written as :

$$2x_1 + 3x_2 \leq 480 \quad \text{———————— (3)}$$

Lastly, there are two more constraints which are not numbered. They are $x_1 \geq 0$ and $x_2 \geq 0$, simply because the quantities x_1 and x_2 cannot be negative.

We can now summarize the problem as a **linear programming model** as follows:

Maximize $Z = 10x_1 + 8x_2$ ———————— (0)

subject to

$5x_1 + 3x_2 \leq 750$ ———————— (1)

$6x_1 + 4x_2 \leq 800$ ———————— (2)

$2x_1 + 3x_2 \leq 480$ ———————— (3)

$x_1 \geq 0$

$x_2 \geq 0$

1.2 Solving a Linear Programming Problem

There are two methods in solving linear programming models, namely, the **graphical method** and the **simplex method**. The graphical method can only solve linear programming problems with two decision variables, while the simplex method can solve problems with any number of decision variables. Since this book will only concentrate on the applications of linear programming, the mathematical details for solving the models will not be thoroughly treated. In this section, the graphical method and the simplex method will only be briefly described.

1.2.1 Graphical Method

Let us look at the linear programming model for Example 1.1:

Max $Z = 10x_1 + 8x_2$ ——— (0)

subject to

$5x_1 + 3x_2 \leq 750$ ——— (1)

$6x_1 + 4x_2 \leq 800$ ——— (2)

$2x_1 + 3x_2 \leq 480$ ——— (3)

$x_1 \geq 0$

$x_2 \geq 0$

The area bounded by (1) : $5x_1 + 3x_2 = 750$, (2) : $6x_1 + 4x_2 = 800$, (3) : $2x_1 + 3x_2 = 480$, (4) : $x_1 = 0$ and (5) : $x_2 = 0$ is called the feasible space, which is the shaded area shown in Fig. 1.1. Any point that lies within this feasible space will satisfy all the constraints and is called a feasible solution.

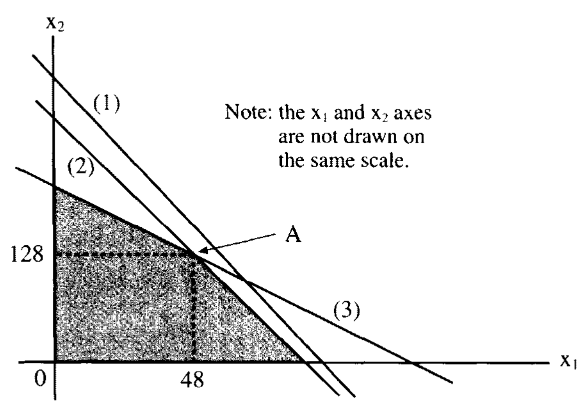

Fig. 1.1 Graphical Method

The optimal solution is a feasible solution which, on top of satisfying all constraints, also optimizes the objective function, that is, maximizes profit in this case. By using the slope of the objective function, -(10/8) in our case, a line can be drawn with such a slope which touches a point within the feasible space and is as far away as possible from the point of origin 0. This point is represented by A in Fig. 1.1 and is the optimal solution. From the graph, it can be seen that at optimum,

x_1 = 48 (type I pipes)

x_2 = 128 (type II pipes)

max Z = 1504 (profit in $), calculated from 10(48) + 8(128)

From Fig. 1.1, one can also see whether or not the resources (i.e. storage space, raw materials, working time) are fully utilized.

Consider the storage space constraint (1). The optimal point A does not lie on line (1) and therefore does not satisfy the equation $5x_1 + 3x_2 = 750$. If we substitute $x_1 = 48$ and $x_2 = 128$ into this equation, we obtain:

$$5(48) + 3(128) = 624 < 750$$

Therefore, at optimum, only 624 m^2 of storage space are used and 126 m^2 (i.e. 750 - 624) are not used.

By similar reasoning, we can see that the other two resources (raw materials and working time) are fully utilized.

If constraint (1) of the above problem is changed to $5x_1 + 3x_2 \leq 624$, that is, the available storage space is 624 m^2 instead of 750 m^2, then line (1) will also touch the feasible space at point A. In this case, lines (1), (2) and (3) are concurrent at point A and all the three resources are fully utilized when the maximum profit is attained. There is a technical term called "optimal degenerate solution" used for such a situation.

1.2.2 Simplex Method

When there are three or more decision variables in a linear programming model, the graphical method is no more suitable for solving the model. Instead of the graphical method, the **simplex method** will be used.

As mentioned earlier, the main theme of this book is **applications** of linear programming, not mathematical theory behind linear programming. Therefore, no detail description of the mathematics of linear programming will be presented here. There are many well developed computer programs available in the market for solving linear programming models using the simplex method. One of them is QSB^+ (Quantitative Systems for Business Plus) written by Y.L. Chang and R.S. Sullivan and can be obtained in any large bookshop world-wide. The author will use the QSB^+ software to solve all the problems contained in the later chapters of this book.

Examples of the techniques employed in the simplex method will be illustrated in Appendix A at the end of this book. Some salient points of the method are summarized below.

First of all we introduce slack variables S_1, S_2 and S_3 ($S_1, S_2, S_3 \geq 0$) for Example 1.1 to change the constraints from inequalities to equalities such that the model becomes:

$$Z - 10x_1 - 8x_2 = 0 \quad \text{(0a)}$$

subject to

$$5x_1 + 3x_2 + S_1 = 750 \quad \text{(1a)}$$

$$6x_1 + 4x_2 + S_2 = 800 \quad \text{(2a)}$$

$$2x_1 + 3x_2 + S_3 = 480 \quad \text{(3a)}$$

The **initial tableau** of the simplex method is shown in Table 1.1. It is in fact a rewrite of equations (0a), (1a), (2a) and (3a) in a tableau format.

Basic Variable	Z	x_1	x_2	S_1	S_2	S_3	RHS
(0a) Z	1	-10	-8	0	0	0	0
(1a) S_1	0	5	3	1	0	0	750
(2a) S_2	0	6	4	0	1	0	800
(3a) S_3	0	2	3	0	0	1	480

Table 1.1 Initial Simplex Tableau for Example 1.1

After two iterations (see Appendix A), the **final tableau** will be obtained and is shown in Table 1.2 below.

Basic Variable		Z	x_1	x_2	S_1	S_2	S_3	RHS
(0c)	Z	1	0	0	0	1.4	0.8	1504
(1c)	S_1	0	0	0	1	-0.9	-0.2	126
(2c)	x_1	0	1	0	0	0.3	-0.4	48
(3c)	x_2	0	0	1	0	-0.2	0.6	128

Table 1.2 Final Simplex Tableau for Example 1.1

To obtain a solution from a simplex tableau, the basic variables are equal to the values in the RHS column. The non-basic variables (i.e. the decision variables or slack variables which are not in the basic variable column) are assigned the value zero. Therefore, from the final tableau, we can see that the optimal solution is:

$Z = 1504$

$x_1 = 48$

$x_2 = 128$

$S_1 = 126$

$S_2 = 0$

$S_3 = 0$ } non-basic variables are equal to 0.

It can be seen that this result is the same as that found by the graphical method. S_1 here is 126, which means that the slack variable for storage space is 126 and therefore 126 m^2 of storage space is not utilized. S_1 and S_2 are slack variables for the other two resources and are equal to 0. This means that the raw materials and the working time are fully utilized.

1.2.3 Revised Simplex Method

The **revised simplex method** is also called the modified simplex method. In this method, the objective function is usually written in the last row instead of the first. Examples of the technique are illustrated in Appendix B. QSB^+ uses the revised simplex method in solving linear programming models. The initial tableau for Example 1.1 is shown in Table 1.3.

Basic Variable	C_j	x_1 10	x_2 8	S_1 0	S_2 0	S_3 0	RHS
S_1	0	5	3	1	0	0	750
S_2	0	6	4	0	1	0	800
S_3	0	2	3	0	0	1	480
Z_j		0	0	0	0	0	0
$C_j - Z_j$		10	8	0	0	0	

Table 1.3 Initial Simplex Tableau for Example 1.1 (Revised Simplex Method)

After two iterations (see Appendix B), the final tableau will be obtained. It is shown in Table 1.4.

Basic Variable	C_j	x_1 10	x_2 8	S_1 0	S_2 0	S_3 0	RHS
S_1	0	0	0	1	-0.9	0.2	126
x_1	10	1	0	0	0.3	-0.4	48
x_2	8	0	1	0	-0.2	0.6	128
Z_j		10	8	0	1.4	0.8	1504
$C_j - Z_j$		0	0	0	-1.4	-0.8	

Table 1.4 Final Simplex Tableau for Example 1.1 (Revised Simplex Method)

2 PRIMAL AND DUAL MODELS

2.1 Shadow Price / Opportunity Cost

The shadow price (or called opportunity cost) of a resource is defined as the economic value (increase in profit) of an extra unit of resource at the optimal point. For example, the raw material available in Example 1.1 of Chapter 1 is 800 kg; the shadow price of it means the increase in profit (or the increase in Z, the objective function) if the raw material is increased by one unit, to 801 kg.

Now, let y_1 = shadow price of storage space ($/m^2)

y_2 = shadow price of raw material ($/kg)

y_3 = shadow price of working time ($/minute)

This means that one additional m^2 of storage space available (i.e. 751 m^2 is available instead of 750 m^2) will increase Z by y_1 dollars; one additional kg of raw materials available will increase Z by y_2 dollars; and one additional minute of working time available will increase Z by y_3 dollars.

Based on the definition of shadow price, we can formulate another linear programming model for Example 1.1. This new model is called the **dual model**.

2.2 The Dual Model

Since the production of a type I pipe requires 5 m^2 of storage space, 6 kg of raw material and 2 minutes of working time, the shadow price of producing one extra type I pipe will be $5y_1 + 6y_2 + 2y_3$. This means that the increase in profit (i.e. Z) due to producing an additional type I pipe is $5y_1 + 6y_2 + 2y_3$, which should be greater than or at least equal to $10, the profit level of selling one type I pipe, in order to justify the extra production. Hence, we can write the constraint that:

$$5y_1 + 6y_2 + 2y_3 \geq 10 \qquad \text{——————} \qquad (1)$$

A similar argument applies to type II pipe, and we can write another constraint that:

$$3y_1 + 4y_2 + 3y_3 \geq 8 \qquad \text{——————} \qquad (2)$$

It is impossible to have a decrease in profit due to an extra input of any resources. Therefore the shadow prices cannot have negative values. So, we can also write:

$y_1 \geq 0$

$y_2 \geq 0$

$y_3 \geq 0$

We can also interpret the shadow price as the amount of money that the pipe company can afford to pay for one additional unit of resource so that he can just break even on the use of that resource. In other words, the company can afford, to pay, say, y_2 for one extra kg of raw material. If it pays less than y_2 from the market to buy the raw material it will make a profit, and vice versa.

The objective this time is to minimize cost. The total price, P, of the total resources employed in producing pipes is equal to $750y_1 + 800y_2 + 480y_3$. In order to minimize P, the objective function is written as:

$$\text{Minimize } P = 750y_1 + 800y_2 + 480y_3 \qquad (0)$$

We can now summarize the **dual model** as follows:

$$\text{Min } P = 750y_1 + 800y_2 + 480y_3 \qquad (0)$$

subject to

$$5y_1 + 6y_2 + 2y_3 \geq 10 \qquad (1)$$

$$3y_1 + 4y_2 + 3y_3 \geq 8 \qquad (2)$$

$y_1 \geq 0$

$y_2 \geq 0$

$y_3 \geq 0$

The solution of this dual model (see Appendix A or Appendix B) is:

min P = 1504

$y_1 = 0$

$y_2 = 1.4$

$y_3 = 0.8$

We can observe that the optimal value of P is equal to the optimal value of Z found in Chapter 1. The shadow price of storage space, y_1, is equal to 0. This means that one additional m^2 of storage space will result in no increase in profit.

This is reasonable because there has already been unutilized storage space. The shadow price of raw material, y_2, is equal to 1.4. This means that one additional kg of raw material will increase the profit level by $1.4. The shadow price of working time, y_3, is equal to 0.8. This means that one additional minute of working time will increase the profit level by $0.8. It can also be interpreted that $0.8/minute is the amount which the company can afford to pay for the extra working time. If the company pays less than $0.8/minute for the workers it will make a profit, and vice versa.

2.3 Comparing Primal and Dual

The linear programming model given in Chapter 1 is referred to as a **primal model**. Its dual form has been discussed in Section 2.2. These two models are reproduced below for easy reference.

Primal	Dual
Max $Z = 10x_1 + 8x_2$	Min $P = 750y_1 + 800y_2 + 480y_3$
subject to	subject to
$5x_1 + 3x_2 \leq 750$	$5y_1 + 6y_2 + 2y_3 \geq 10$
$6x_1 + 4x_2 \leq 800$	$3y_1 + 4y_2 + 3y_3 \geq 8$
$2x_1 + 3x_2 \leq 480$	$y_1 \geq 0$
$x_1 \geq 0$	$y_2 \geq 0$
$x_2 \geq 0$	$y_3 \geq 0$

It can be observed that :

(a) the coefficients of the objective function in the primal model are equal to the RHS constants of the constraints in the dual model,

(b) the RHS constants of the constraints of the primal model are the coefficient of the objective function of the dual model, and

(c) the coefficients of y_1, y_2 and y_3, when read row by row, for the two constraints of the dual model are equal to those of x_1 and x_2, when read column by column, in the primal model. In other words, the dual is the transpose of the primal if the coefficients of the constraints are imagined as a matrix.

The general form of a primal model is :

$$\text{Max } Z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$$

subject to

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \leq b_2 \\ \quad \vdots \qquad\qquad\qquad\qquad\qquad\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n \leq b_m \\ \qquad\qquad\qquad\qquad \text{all } x_j \geq 0 \end{array}$$

The general form of the dual model will be :

$$\text{Min } P = b_1y_1 + b_2y_2 + \ldots\ldots + b_my_m$$

subject to

$$\begin{array}{l} a_{11}y_1 + a_{21}y_2 + \ldots\ldots + a_{m1}y_m \geq c_1 \\ a_{12}y_1 + a_{22}y_2 + \ldots\ldots + a_{m2}y_m \geq c_2 \\ \quad \vdots \qquad\qquad\qquad\qquad\qquad\quad \vdots \\ a_{1n}y_1 + a_{2n}y_2 + \ldots\ldots + a_{mn}y_m \geq c_n \\ \qquad\qquad\qquad\qquad \text{all } y_i \geq 0 \end{array}$$

The two models are related by :

(a) maximum Z = minimum P, and

(b) $$y_i = \frac{\Delta Z}{\Delta b_i}$$

where y_i stand for the shadow price of the resource i and b_i is the amount of the i^{th} resource available.

It should be pointed out that it is not necessary to solve the dual model in order to find y_i. In fact, y_i can be seen from the final simplex tableau of the primal model. Let us examine the final tableau of Example 1.1 :

Basic Variable	Z	x_1	x_2	S_1 (y_1)	S_2 (y_2)	S_3 (y_3)	RHS
Z	1	0	0	0	1.4	0.8	1504
S_1	0	0	0	1	-0.9	-0.2	126
x_1	0	1	0	0	0.3	-0.4	48
x_2	0	0	1	0	-0.2	0.6	128

We can see that the coefficient of S_1, slack variable for storage space, in the first row (the row of the objective function Z) is equal to y_1, the shadow price of storage space in $/m^2. The coefficient of S_2, slack variable for raw material, in the first row is equal to y_2, the shadow price of raw material in $/kg. Again, the coefficient of S_3, slack variable for working time, in the first row is equal to y_3, the shadow price of working time in $/minute.

Therefore, it is not necessary to solve the dual model to find the values of the decision variables (ie. y_1, y_2 and y_3). They can be found from the primal model. The same occurs in the revised simplex method. The final tableau of the revised method for Example 1.1 is :

Basic		x_1	x_2	S_1	S_2	S_3	
Variable	C_j	10	8	0	0	0	RHS
S_1	0	0	0	1	-0.9	0.2	126
x_1	10	1	0	0	0.3	-0.4	48
x_2	8	0	1	0	-0.2	0.6	128
Z_j		10	8	0 (y_1)	1.4 (y_2)	0.8 (y_3)	1504
$C_j - Z_j$		0	0	0	-1.4	-0.8	

In a similar way, the values of y_1, y_2 and y_3 can be seen from the row of Z_j.

2.4 Algebraic Way to Find Shadow Prices

There is a simple algebraic way to find the shadow price without employing the simplex method. Let us use the same example, Example 1.1, again to illustrate how this can be done. The linear programming model is reproduced hereunder for easy reference:

$$\text{Max} \quad Z = 10x_1 + 8x_2 \qquad \text{———} \qquad (0)$$

subject to

$$5x_1 + 3x_2 \leq 750 \qquad \text{(1)}$$

$$6x_1 + 4x_2 \leq 800 \qquad \text{(2)}$$

$$2x_1 + 3x_2 \leq 480 \qquad \text{(3)}$$

As can be seen from the graphical method, storage space has no effect on the optimal solution (see Section 1.2.1). Line (1) : $5x_1 + 3x_2 = 750$ therefore does not pass through the optimal point. Since raw material and working time both define the optimal point, the solution is therefore the intersection point of line (2) : $6x_1 + 4x_2 = 800$ and line (3) : $2x_1 + 3x_2 = 480$.

Assuming that the raw material available is increased by ΔL kg, the optimal solution will then be obtained by solving:

$$6x_1 + 4x_2 = 800 + \Delta L$$

and $$2x_1 + 3x_2 = 480$$

Solving, we obtain :

$$x_1 = 48 + \frac{3}{10}\Delta L$$

and $$x_2 = 128 - \frac{1}{5}\Delta L$$

Substituting x_1 and x_2 into the objective function, we have:

$$Z + \Delta Z = 10\left(48 + \frac{3}{10}\Delta L\right) + 8\left(128 - \frac{1}{5}\Delta L\right)$$

Simplifying, we get

$$Z + \Delta Z = 10(48) + 8(128) + \frac{7}{5}\Delta L$$

Since $Z = 10(48) + 8(128)$

$$\therefore \Delta Z = \frac{7}{5}\Delta L$$

i.e. $\dfrac{\Delta Z}{\Delta L} = 1.4$ (i.e. shadow price of raw material in \$/kg)

Similarly, if we assume that the working time available is increased by ΔM minutes, we can, by the same method, obtain that :

$$\frac{\Delta Z}{\Delta M} = 0.8 \qquad \text{(i.e. shadow price of working time in \$/minute)}$$

2.5 A Worked Example

Let us now see a practical example of the application of shadow prices.

Example 2.1

A company which manufactures table lamps has developed three models denoted the "Standard", "Special" and "Deluxe". The financial returns from the three models are \$30, \$40 and \$50 respectively per unit produced and sold. The resource requirements per unit manufactured and the total capacity of resources available are given below :

	Machining (hours)	Assembly (hours)	Painting (hours)
Standard	3	2	1
Special	4	2	2
Deluxe	4	3	3
Available Capacity	20,000	10,000	6,000

(a) Find the number of units of each type of lamp that should be produced such that the total financial return is maximized. (Assume all units produced are also sold.)

(b) At the optimal product mix, which resource is under-utilized?

(c) If the painting-hours resource can be increased to 6,500, what will be the effect on the total financial return?

Solution 2.1

(a) The problem can be represented by the following linear programming model :

Max $Z = 30x_1 + 40x_2 + 50x_3$

subject to

$3x_1 + 4x_2 + 4x_3 \leq 20{,}000$

$2x_1 + 2x_2 + 3x_3 \leq 10{,}000$

$x_1 + 2x_2 + 3x_3 \leq 6{,}000$

$x_1, x_2, x_3 \geq 0$

where x_1 = number of Standard lamps produced

x_2 = number of Special lamps produced

x_3 = number of Deluxe lamps produced

Introduce slack variables S_1, S_2 and S_3 such that:

$3x_1 + 4x_2 + 4x_3 + S_1 = 20{,}000$

$2x_1 + 2x_2 + 3x_3 + S_2 = 10{,}000$

$x_1 + 2x_2 + 3x_3 + S_3 = 6{,}000$

Using the simplex method to solve the model, the final tableau is:

shadow price of painting hour

Basic Variable	Z	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	1	0	0	10	0	10	10	160,000
S_1	0	0	0	-2	1	-1	-1	4,000
x_1	0	1	0	0	0	1	-1	4,000
x_2	0	0	1	1.5	0	-0.5	1	1,000

The final tableau of the revised simplex method is :

Basic Variable		x_1	x_2	x_3	S_1	S_2	S_3	
	C_j	30	40	50	0	0	0	RHS
S_1	0	0	0	-2	1	-1	-1	4,000
x_1	30	1	0	0	0	1	-1	4,000
x_2	40	0	1	1.5	0	-0.5	1	1,000
Z_j		30	40	60	0	10	10	160,000
$C_j - Z_j$		0	0	-10	0	-10	-10	

shadow price of painting hour

From the final tableau of either method, we obtain the following optimal solution:

Basic variable	Non-basic variable
S_1 = 4,000	x_3 = 0
x_1 = 4,000	S_2 = 0
x_2 = 1,000	S_3 = 0
Z = 160,000	

Therefore, the company should manufacture 4,000 Standard lamps, 1,000 Special lamps and no Deluxe lamps. The maximum financial return is $160,000.

(b) Machining is under-utilized. Since S_1 = 4,000, therefore 4,000 machining hours are not used.

(c) The shadow price of painting-hours resource can be read from either tableau and is equal to $10/hour. This means that the financial return will increase by $10 if the painting-hour resource is increased by 1 hour. If the painting hour is increased by 500 (from 6,000 hours to 6,500 hours), then the total increase in financial return is $10 x 500 = $5,000, and the overall financial return will be $160,000 + $5,000 = $165,000.

It should be noted that the shadow price $10/per hour may not be valid for infinite increase of painting hours. In this particular problem, it is valid until the painting hour is increased to 10,000. This **involves post optimality analysis** which is outside the scope of this book. The software QSB, however, shows the user the range of validity for each and every resource in its sensitivity analysis function.

It is also worthwhile to note that while the Z_j values of the final revised simplex tableau represent shadow prices of the corresponding resources, the C_j - Z_j values represent the **reduced costs**. The reduced cost of a non-basic decision variable is the reduction in profit of the objective function due to one unit increase of that non-basic decision variable. In Example 2.1, the non-basic decision variable is x_3, and so x_3 is equal to 0 when the total profit is maximized to $160,000. If we want to produce one Deluxe lamp in the product mix (i.e. $x_3 = 1$) under the original available resources, then the total profit will be reduced to $159,990 (i.e. 160,000 - 10) because the reduced cost of x_3 is -10 (the $C_j - Z_j$ value under column x_3).

Exercise

1. A precast concrete subcontractor makes three types of panels. In the production the quantities of cement, coarse aggregates and fines aggregates required are as follows:

	Cement (m^3/panel)	Course aggregates (m^3/panel)	Fines aggregates (m^3/panel)
Panel I	1	3	2
Panel II	1	2	3
Panel III	2	3	4

The subcontractor has the following quantities of cement, coarse aggregates and fines aggregates per week :

Cement : 300 m^3

Coarse aggregates : 500 m^3

Fines aggregates : 620 m^3

The financial return for panel types I, II and III are 20, 18 and 25 respectively. Find the number of panels of each type that should be made so that the total financial return is maximized. Which resource is under-utilized and why? If the subcontractor can obtain some extra coarse aggregates, what is the maximum cost per m^3 the subcontractor can afford to pay for it?